

$$10 a) y = D e^{\int -x dx} = D e^{-\frac{1}{2}x^2} \quad (2P)$$

$$-2 = y(0) = D \quad (2P)$$

$$b) y = D e^{\int 5x \cos x dx} = D e^{5 \sin x + 10 \cos x} \quad (2P)$$

$$1 = y(0) = D e^1 \Rightarrow D = \frac{1}{e} \quad (2P)$$

$$c) y = D e^{-\int \frac{dx}{1+x}} = D e^{-\ln|1+x|} = \frac{D}{1+x} \quad (2P)$$

$$-1 = y(0) = D \quad (2P)$$

$$d) y = D e^{\int \frac{1}{4-x^2} dx} = D e^{\frac{1}{4} \left(\int \frac{1}{2-x} + \frac{1}{2+x} dx \right)} = D e^{\frac{1}{4} \left(-\ln|2-x| + \ln|2+x| \right)} = D \left(\frac{2+x}{2-x} \right)^{\frac{1}{4}} \quad (2P)$$

$$0 = y(0) = D$$

$$e) y = D e^{\int \frac{1}{x^2+2x+2} dx} = D e^{\int \frac{1}{(x+1)^2+1} dx}$$

$$= D e^{\arctan(x+1)} \quad (2P)$$

$$2 = y(0) = D e^{\frac{\pi}{4}}, \quad D = 2e^{-\frac{\pi}{4}} \quad (2P)$$

$$11 a) \text{ Integrating factor } u(x) = e^{-\int 3 dx} = e^{-3x}$$

$$\frac{d}{dx}(e^{-3x} y) = 5e^{-3x} \Rightarrow e^{-3x} y = -\frac{5}{3} e^{-3x} + C_1 \quad (4P)$$

$$\Rightarrow y = C_1 e^{3x} + \frac{5}{3} \quad (4P)$$

$$-2 = y(0) = C_1 + \frac{5}{3} \Rightarrow C_1 = -\frac{1}{3} \quad (4P)$$

$$b) u(x) = e^{\int 2x dx} = e^{x^2} \quad (4P)$$

$$\Rightarrow \frac{d}{dx}(e^{x^2} y) = e^{x^2} \cdot 2x \Rightarrow e^{x^2} y = e^{x^2} + C_1$$

$$\Rightarrow y = C_1 e^{-x^2} + 1 \quad (4P)$$

$$1 = y(0) = C_1 + 1 \Rightarrow C_1 = 0 \quad (4P)$$

$$c) u(x) = e^{-\int \frac{3x^2}{1+x^3} dx} = e^{-\ln|1+x^3|} = \frac{1}{1+x^3} \quad (4P)$$

$$\frac{d}{dx} \left(\frac{y}{1+x^3} \right) = \frac{x^2+x^5}{1+x^3} = x^2 \Rightarrow \frac{y}{1+x^3} = \frac{1}{3} x^3 + C_1$$

$$\Rightarrow y = C_1 (1+x^3) + \frac{1}{3} x^3 (1+x^3) \quad (4P)$$

$$-1 = y(0) = C_1 \quad (4P)$$

$$d) u(x) = e^{-\int 5 dx} = e^{-5x} \quad (4P)$$

$$\frac{d}{dx}(e^{-5x} y) = e^{-5x} \cdot 2x e^{2x} = 2x e^{-3x} \Rightarrow e^{-5x} y = 2x e^{-3x} - 2e^{-3x} + C_1$$

$$\Rightarrow y = C_1 e^{5x} + (2x-2)e^{2x} \quad (4P)$$

$$3 = y(0) = C_1 - 2 \Rightarrow C_1 = 5 \quad (4P)$$

$$e) u(x) = e^{\int \tan x dx} = e^{-\ln|\cos x|} = \frac{1}{\cos x} \quad (4P)$$

$$\frac{d}{dx} \left(\frac{1}{\cos x} \cdot y \right) = \frac{1}{\cos x} \cdot \cos x \Rightarrow \frac{1}{\cos x} y = x + C_1$$

$$\Rightarrow y = C_1 \cdot \cos(x) + x \cos x \quad (4P)$$

$$2 = y(0) = C_1 \quad (4P)$$

$$12) a) \quad z = \frac{1}{y^2}$$

$$\Rightarrow z' = -2 \frac{y'}{y^3} = -2 \frac{y' - x y^3}{y^3} = -2 \frac{1}{y^2} + 2x = -2z + 2x \quad (10P)$$

b) Integrating factor $u(x) = e^{\int 2x dx} = e^{x^2}$ (2P)

$$\frac{d}{dx}(z e^{x^2}) = 2x e^{x^2} \Rightarrow z e^{x^2} = x e^{x^2} + \frac{1}{2} \quad (2P)$$

$$\Rightarrow z = x e^{-x^2} + x - \frac{1}{2} \quad (2P)$$

$$\Rightarrow y = \frac{1}{\sqrt{z}} = \frac{1}{\sqrt{xe^{-x^2} + x - \frac{1}{2}}} \quad (4P)$$

D) $u(x) = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$$\frac{d}{dx}(xy) = x \Rightarrow xy = \frac{1}{2}x^2 + C$$

$$\Rightarrow y = \frac{C}{x} + \frac{1}{2}x \quad C \text{ "general solution"}$$

But if $C \neq 0$ then $y(x)$ is not defined for $x=0$. In fact, the initial condition $y(0)=1$ cannot be fulfilled. (10P)

Reason: The right hand side of the differential equation is singular at $x=0$.
(i.e. right hand side not defined at $x=0$) (10P)